



## COURSE DESCRIPTION CARD - SYLLABUS

Course name

Stochastic processes [S2MwT1>PS]

### Course

Field of study

Mathematics in Technology

Year/Semester

1/1

Area of study (specialization)

–

Profile of study

general academic

Level of study

second-cycle

Course offered in

polish

Form of study

full-time

Requirements

compulsory

### Number of hours

Lecture

30

Laboratory classes

0

Other (e.g. online)

0

Tutorials

30

Projects/seminars

0

### Number of credit points

4,00

### Coordinators

dr Kamil Świątek

### Lecturers

### Prerequisites

A student knows the basic concepts of the probability theory. The student applies appropriate theorems to determine the probability of random events, determines the parameters of random variables of discrete and continuous type, models random phenomena with the use of random variable distributions. The student understands the need for further education.

### Course objective

The main aim of this course is to acquaint the student with the basic concepts of the stochastic processes theory and certain classes of stochastic processes, and to gain by the student the ability to calculate some characteristics of those processes. Furthermore the student will get to know the notion of stochastic integral, will get to acquire the ability to determine the stochastic integrals and the stochastic differentials (stochastic calculus).

### Course-related learning outcomes

Knowledge:

1. Student has knowledge of the stochastic processes theory concerning the possibility of applying selected types of stochastic processes to the modeling of relevant random phenomena.
2. Student knows the basic concepts and theorems of the stochastic processes theory, and the examples

of selected classes of stochastic processes.

**Skills:**

Student determines the characteristics of stochastic processes, verifies the belonging of a given stochastic process to the appropriate class of stochastic processes, determines stochastic integrals and differentials, uses the appropriate types of stochastic processes to the modeling of relevant random phenomena.

**Social competences:**

1. Student is aware of the existence of random factors affecting the modeled phenomenon.
2. Student is aware of the role and importance of knowledge in solving practical problems.
3. Student is aware of the need to deepen and expand knowledge.

**Methods for verifying learning outcomes and assessment criteria**

Learning outcomes presented above are verified as follows:

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**Lecture:**

Knowledge acquired during the lecture is verified on the basis of an exam. To pass the lecture it is necessary to get at least 50% of the points from the mentioned exam.

**Tutorials:**

Skills acquired during the classes are verified on the basis of two colloquia. To pass the classes it is necessary to get at least 50% of the total number of points from the mentioned colloquia.

**Grading system:**

- 0%-50% - 2.0,
- 50%-60% - 3.0,
- 60%-70% - 3.5,
- 70%-80% - 4.0,
- 80%-90% - 4.5,
- 90%-100% - 5.0.

**Programme content**

**Lecture:**

1. Basic concepts of the stochastic processes theory (a stochastic process, a trajectory of a stochastic process, real-life phenomena as examples of stochastic processes, a modification of a stochastic process, an indistinguishability of stochastic processes).
2. Probabilistic description of a stochastic process (characteristics of a stochastic process, an uncorrelated stochastic processes, finite-dimensional distributions of a stochastic process, the Kolmogorov existence theorem, an independence of stochastic processes, a stationarity of a stochastic process, a Gaussian process).
3. Canonical representation of a stochastic process (transformation of a given stochastic process to its canonical representation, approximation of a given stochastic process by the use of its canonical representation).
4. Stochastic processes with uncorrelated increments and stochastic processes with independent increments (an increment of a stochastic process, definition of a stochastic process with uncorrelated increments, definition of a stochastic process with independent increments, an homogeneous Poisson process and its properties).
5. Telegraphic signals (synchronous and asynchronous stochastic processes).
6. Markov process (definition of a Markov process, a Markov chain, an homogeneous Markov process, a stationary Markov chain, the Chapman–Kolmogorov equation, the Kolmogorov differential equation system).
7. Differentiability and integrability of a stochastic process in the mean square sense (a continuity in the mean square sense, a derivative in the mean square sense, a stochastic integral in the mean square sense, an ergodicity of a stochastic process).
8. Elements of spectral analysis of stationary stochastic processes (a spectral density of a stochastic process, a cross-spectral density of stochastic processes).
9. Martingales and stopping times (a filtration, definition of a martingale and its properties, a sub-martingale and a super-martingale, definition of a stopping time and its properties).
10. Wiener process (definition of a Wiener process and its properties, the Kolmogorov continuity

theorem).

11. Itô stochastic integral (construction of a Itô stochastic integral and its properties, Itô's process, Itô's stochastic differential, Itô's lemma).

Tutorials:

1. Characteristics of stochastic processes in analytic representation.
2. Characteristics of sums and products of stochastic processes.
3. Canonical representation of a stochastic process.
4. Examples of stochastic processes with independent increments.
5. Application of stochastic processes to modelling of telegraphic signals.
6. Examples of Markov processes.
7. Selected exercises related to the mass service problems.
8. Characteristics of stochastic processes differentiable and integrable in the mean square sense.
9. Elements of spectral analysis of stationary stochastic processes.
10. Martingales and stopping times.
11. Wiener process.
12. Application of Itô's formula to determining of Itô's stochastic integrals and Itô's stochastic differentials.

### Teaching methods

Lecture: traditional lecture (theory presented in connection with the current knowledge of students).

Tutorials: blackboard tutorials (solving of math problems with the help of a teacher).

### Bibliography

Basic

1. A. Plucińska, E. Pluciński, Probabilistyka: statystyka matematyczna, procesy stochastyczne, rachunek prawdopodobieństwa, Wydawnictwo Naukowe PWN SA, Warszawa 2017.
2. M. Matalytski, O. Tikhonenko, Procesy stochastyczne, Akademicka Oficyna Wydawnicza EXIT, Warszawa 2011.
3. A. Pieniążek, J. Weiss, A. Winiarz, Procesy stochastyczne w problemach i zadaniach, Wydaw. Politechniki Krakowskiej im. Tadeusza Kościuszki, Kraków 2007.

Additional

1. R. Sz. Lipcer, A. N. Szirajew, Statystyka procesów stochastycznych: filtracja nieliniowa i zagadnienia pokrewne, PWN, Warszawa 1981.
2. A. Iwanik, J. K. Misiewicz, Wykłady z procesów stochastycznych z zadaniami. Cz. 1: Procesy Markowa, Oficyna Wydaw. Uniwersytetu Zielonogórskiego, Zielona Góra 2009.

### Breakdown of average student's workload

|   | Hours | ECTS |
|---|-------|------|
| Total workload  | 100   | 4,00 |
| Classes requiring direct contact with the teacher   | 64    | 3,00 |
| Student's own work (literature studies, preparation for laboratory classes/ tutorials, preparation for tests/exam, project preparation) | 36    | 1,00 |